Further Comments on $G(2)$

One can also look at the problems with various potentials by plotting in two dimensions. I often take

$$q_1 = \frac{s + 2\,t}{7} \;\; ; \;\; q_2 = \frac{2s - \,t}{7} \; . \tag{1}$$

Consider first the perturbative potential,

$$V_{pert}(q) = V_2(q_1 - q_2) + V_2(2q_1 + q_2) + V_2(q_1 + 2q_2) + V_2(q_1) + V_2(q_2) + V_2(q_1 + q_2) \; , \tag{2}$$

where

$$V_2(x) = y^2(1-y)^2 \;\; ; \;\; y = |x|_{mod\,1} \; . \tag{3}$$

To generate a confined vacuum, the maximum of the perturbative potential should be at $s = 1$. Instead, the maximum is at

$$s_c \sim 0.9541424 \; . \tag{4}$$

It is also interesting to plot the potential in both s and t, as in Fig. (1), where

$$V_{pert}\left(\frac{s + 2\,t}{7}, \frac{2s - \,t}{7}\right) \tag{5}$$

is shown, for $s \sim s_c$. As can be seen from Fig. (1), while the perturbative potential is extremal in s about s_c, it is *not* extremal in t.

Consider next the "$G(2)$" non-perturbative potential,

$$-\,V^{G(2)}_{non}(q) = V_1(q_1 - q_2) + V_1(2q_1 + q_2) + V_1(q_1 + 2q_2) + V_1(q_1) + V_1(q_2) + V_1(q_1 + q_2) \; , \tag{6}$$

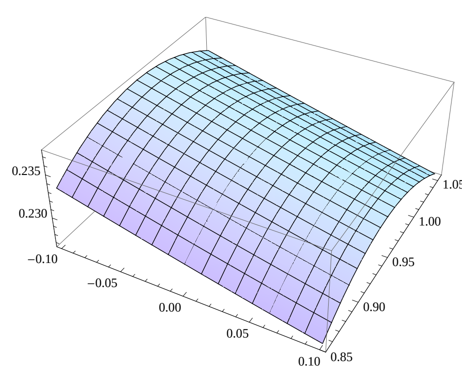

FIG. 1: The perturbative potential for $G(2)$, Eq. (2) plotted about $s = s_c$, and $t = 0$

where

$$V_1(x) = y(1-y) \;\; ; \;\; y = |x|_{mod\, 1} \; . \tag{7}$$

The potential

$$V^{G(2)}_{non}\left(\frac{s+2\,t}{7}, \frac{2s-\,t}{7}\right) \tag{8}$$

is plotted in Fig. (2). This potential is extremal about $s = 1$, but it is not extremal about $t = 0$.

Because the $G(2)$ non-perturbative potential is not extremal in t, the state for $t = 0$ cannot be extremal. This may explain the presence of a non-confined state which was found by Chris and Yun near T_c.

Thus let us consider potentials which are those of an $SU(7)$ gauge group. There are two types: those involving V_1,

$$-V^{SU(7)}_{non,1}(q) \sim V_1(2q_1) + V_1(2q_2) + V_1(2q_1 + 2q_2) +$$

$$+2\left(V_1(q_1 - q_2) + V_1(2q_1 + q_2) + V_1(q_1 + 2q_2)\right) + 4\left(V_1(q_1) + V_1(q_2) + V_1(q_1 + q_2)\right) \; . \tag{9}$$

and that involving V_2,

$$-V^{SU(7)}_{non,2}(q) \sim V_2(2q_1) + V_2(2q_2) + V_2(2q_1 + 2q_2) +$$

$$+2\left(V_2(q_1 - q_2) + V_2(2q_1 + q_2) + V_2(q_1 + 2q_2)\right) + 4\left(V_2(q_1) + V_2(q_2) + V_2(q_1 + q_2)\right) \; . \tag{10}$$

As is shown in Fig. (3) and Fig. (4), each of these two potentials is not only extremal in s about $s = 1$, but is also extremal in t about $t = 0$.

My suggestion for the effective potential of $G(2)$ is then a sum of the perturbative potential in Eq. (2), and the "$SU(7)$" non-perturbative potentials in Eqs. (9) and (10).

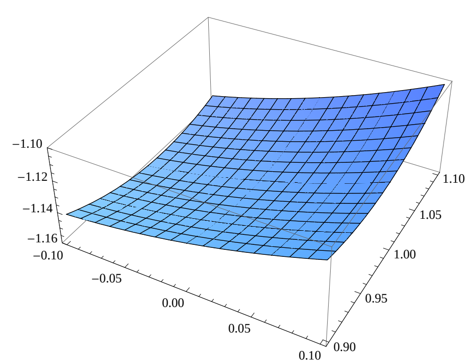

FIG. 2: The non-perturbative potential for $G(2)$, Eq. 6, plotted in s and t, about $s = 1$ and $t = 0$.

Because the perturbative potential is extremal not at $s = 1$, but $s = s_c$, and is not extremal in t, avoiding the presence of non-confined states near T_c is not automatic. However, this will have to be tested through detailed analysis.

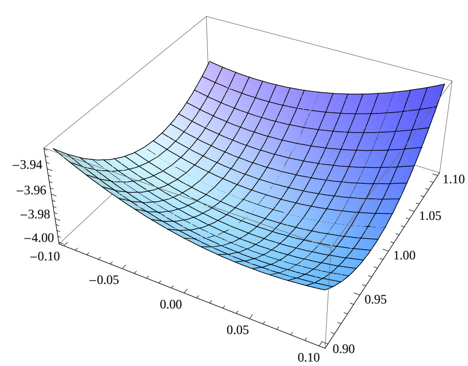

FIG. 3: The "$SU(7)$" non-perturbative potential for Eq. (9), plotted in s and t, about $s = 1$ and $t = 0$.

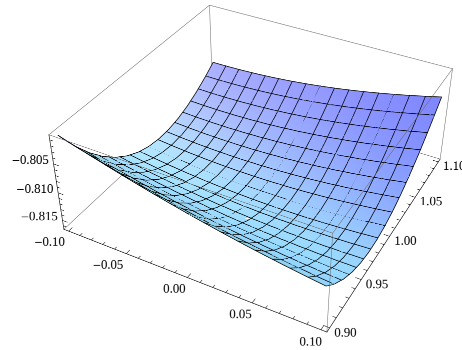

FIG. 4: The "$SU(7)$" non-perturbative potential for Eq. (10), plotted in s and t, about $s = 1$ and $t = 0$.